1. Určete definiční obor (a načrtněte ho) a obor hodnot funkce:

(a) $f(x,y) = \sqrt{1-x^2} + \sqrt{y^2-1}$,

(b) $f(x,y) = \arcsin \frac{y-1}{x}$,

$(x,y) \in \mathbb{R}^2$

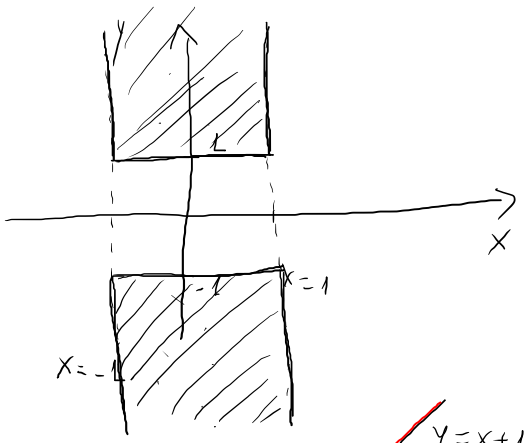
obor hodnot $(f) = \langle 0, \infty)$

a)

$$\begin{cases} 1 - x^2 \geq 0 \\ y^2 - 1 \geq 0 \end{cases}$$

$-1 \leq x \leq 1$

$y \leq -1, \; y \geq 1$

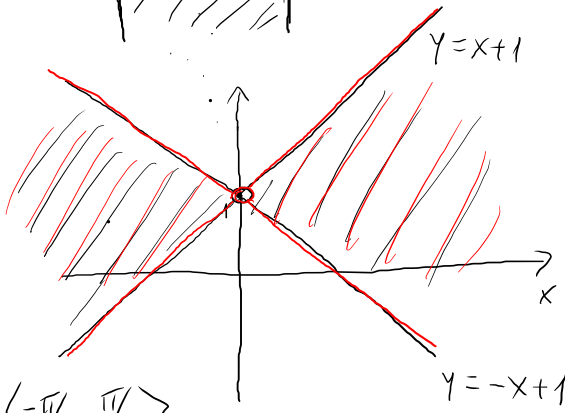


b)

$$-1 \leq \frac{y-1}{x} \leq 1 \qquad x \neq 0$$

$$\begin{cases} \frac{y-1}{x} \geq -1 \\ \frac{y-1}{x} \leq 1 \end{cases} \quad \text{nebo} \quad \begin{cases} -x \leq y-1 \leq x \\ x > 0 \end{cases} \quad \begin{cases} 1-x \leq y \leq x+1 \\ x > 0 \end{cases}$$

$$\begin{cases} -x \geq y-1 \geq x \\ x < 0 \end{cases} \quad \begin{cases} 1-x \geq y \geq x+1 \\ x < 0 \end{cases}$$

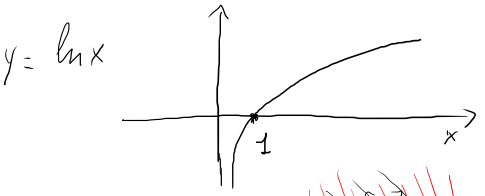


obor hodnot $= \langle -\pi/2, \pi/2 \rangle$

1.4 Určete a načrtněte definiční obory následujících funkcí:

(a) $f(x,y) = \ln\left(x \ln(y-x)\right)$;

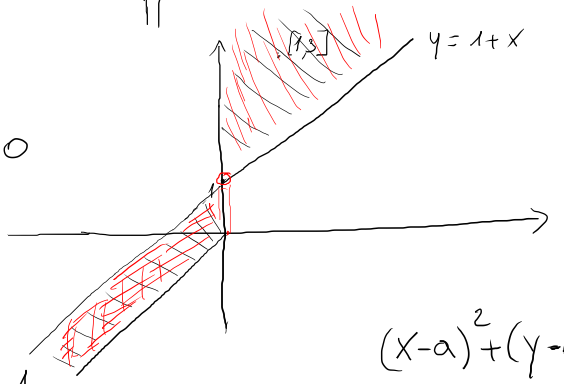
(b) $f(x,y) = \sqrt{\frac{x^2+2x+y^2}{x^2-2x+y^2}}$.



a) $x \cdot \ln(y-x) > 0$

$\begin{cases} x > 0 \\ \ln(y-x) > 0 \end{cases}$ nebo $\begin{cases} x < 0 \\ \ln(y-x) < 0 \end{cases}$

$\begin{cases} x > 0 \\ y - x > 1 \end{cases}$ nebo $\begin{cases} x < 0 \\ 0 < y - x < 1 \end{cases} \iff x < y < 1 + x$



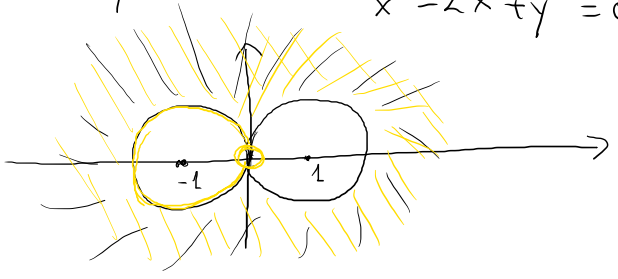
b) $\frac{x^2+2x+y^2}{x^2-2x+y^2} \geq 0$

$x^2 + 2x + y^2 = 0$ (+1, −1)

$(x+1)^2 + y^2 = 1$

$x^2 - 2x + y^2 = 0$

$(x-1)^2 + y^2 = 1$



$\begin{cases} x^2+2x+y^2 \geq 0 \\ x^2-2x+y^2 > 0 \end{cases}$ nebo $\begin{cases} x^2+2x+y^2 < 0 \\ x^2-2x+y^2 < 0 \end{cases}$ $\emptyset$

$(x-a)^2 + (y-b)^2 = R^2$

$A = [a, b]$ $r = R$

Pro následující funkce f vždy načrtněte graf a popište vrstevnice: $c \in \mathbb{R}$ $H_c = \{(x,y) : f(x,y) = c\}$

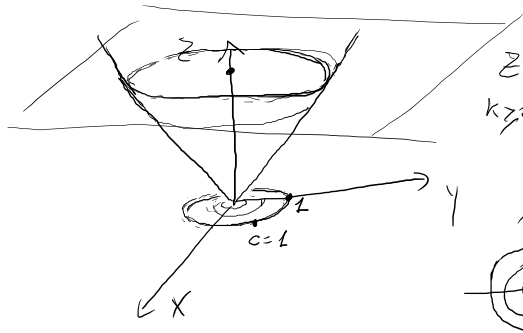
(a) $f(x,y) = \sqrt{x^2+y^2}$ $G(f) = \{(x,y,z) : z = f(x,y)\}$

(b) $f(x,y) = x^2 + 2y^2$

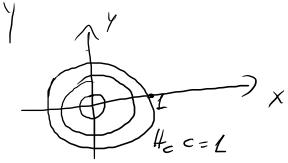
a) $z = \sqrt{x^2+y^2}$

$\begin{cases} z = \sqrt{x^2+y^2} \\ x = 0 \end{cases}$ $\begin{cases} z = \sqrt{x^2+y^2} \\ y = 0 \end{cases}$ $z = |x|$

$z = \sqrt{y^2} = |y|$

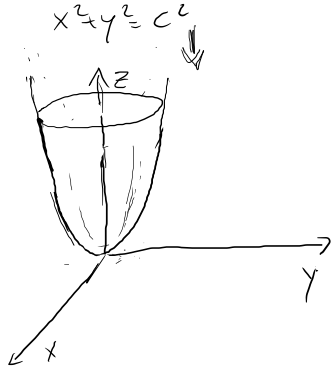


$z = k$ $k \geq 0$ $\sqrt{x^2+y^2} = k$ $x^2+y^2 = k^2$



$c \in \mathbb{R}$ $\sqrt{x^2+y^2} = c$ $c \geq 0$

$x^2+y^2 = c^2$

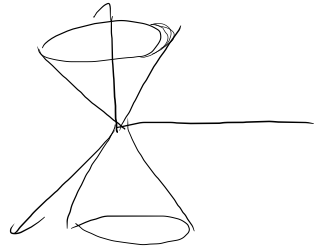


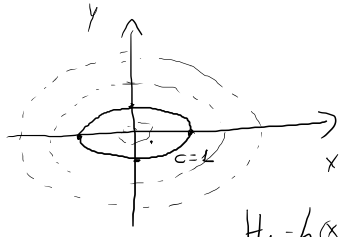
b) $z = x^2 + 2y^2$

$\begin{cases} z = x^2+2y^2 \\ y = 0 \end{cases}$ $z = x^2$

$\begin{cases} z = x^2+2y^2 \\ x = 0 \end{cases}$ $z = 2y^2$

$z^2 = x^2+y^2$





$c = 1$ $H_1 = \{(x,y) : x^2+2y^2 = 1\}$

Pro následující funkce f vždy načrtněte graf a popište vrstevnice:

(e) $f(x,y) = xy$ (hyperbolický paraboloid),

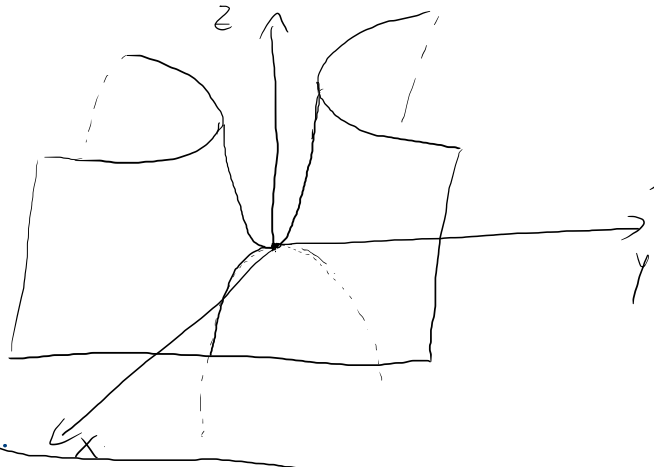
(f) $f(x,y) = x^2 - y^2$ (hyperbolický paraboloid).

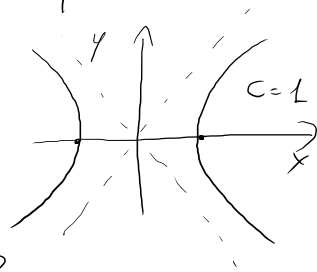
$x^2 - y^2 = c$ $\quad c = 1$

$x^2 - y^2 = 1$

$\begin{cases} z = x^2 - y^2 \\ x = 0 \end{cases}$ $\quad z = -y^2$

$\begin{cases} z = x^2 - y^2 \\ y = 0 \end{cases}$ $\quad z = x^2$



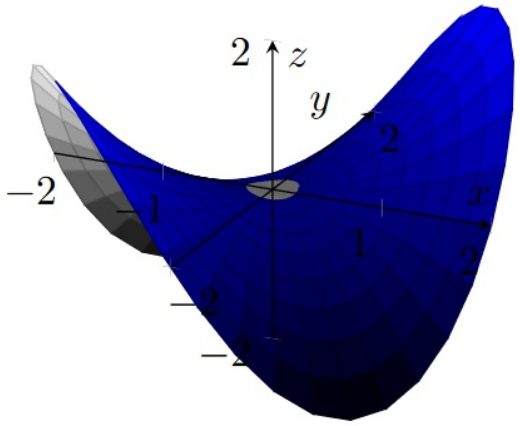


$x = a - b$
$y = a + b$
$x \cdot y = a^2 - b^2$

$\phi: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\pi/4 & -\sin\pi/4 \\ \cos\pi/4 & \sin\pi/4 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$

$= \begin{pmatrix} \frac{\sqrt{2}}{2}(a-b) \\ \frac{\sqrt{2}}{2}(a+b) \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} a-b \\ a+b \end{pmatrix}$

$f \circ \phi = \frac{1}{2}(a^2 - b^2)$



hyperbolický paraboloid (nebo také sedlová plocha)

Určete definiční obor (a načrtněte ho)

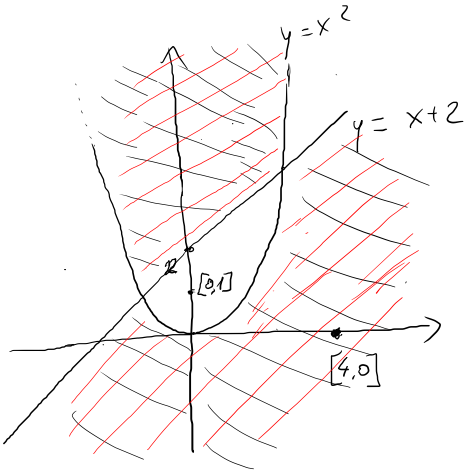
(f) $f(x,y) = \ln \frac{x-y+2}{x^2-y}$,

(g) $f(x,y) = \sqrt{\frac{x^2+y^2-x}{2x-x^2-y^2}}$,

f) $\frac{x-y+2}{x^2-y} > 0$

$\begin{cases} x-y+2 > 0 \\ x^2-y > 0 \end{cases}$ nebo $\begin{cases} x-y+2 < 0 \\ x^2-y < 0 \end{cases}$

$\begin{cases} y < x+2 \\ y < x^2 \end{cases}$ $\begin{cases} y > x+2 \\ y > x^2 \end{cases}$



g) $\frac{x^2+y^2-x}{2x-x^2-y^2} \geq 0$

$x^2+y^2-x = 0$

$(x-1/2)^2 + y^2 = 1/4$

$C_2 = [1/2, 0]$ $r = 1/2$

$2x-x^2-y^2 = 0$

$x^2-2x+y^2 = 0$

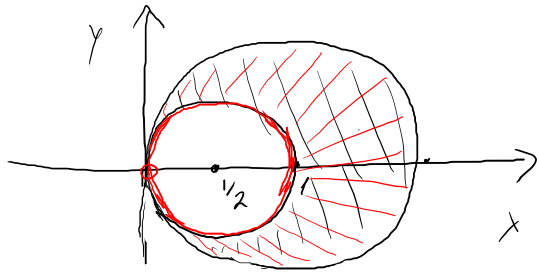
$(x-1)^2+y^2 = 1$

$C_1 = [1,0]$ $r = 1$

$\begin{cases} x^2+y^2-x \geq 0 \\ 2x-x^2-y^2 > 0 \Leftrightarrow x^2-2x+y^2 < 0 \end{cases}$

nebo

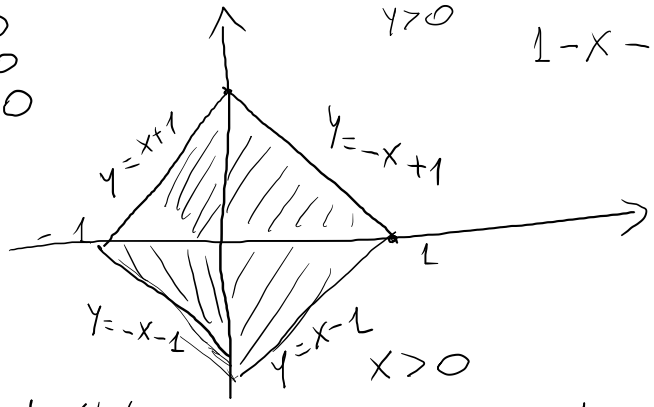
$\begin{cases} x^2+y^2-x < 0 \\ 2x-x^2-y^2 < 0 \end{cases}$ $\emptyset$ $x^2+y^2-2x > 0$



$g(x,y) = \sqrt{1-|x|-|y|}$

$D(g) = ?$

$1-|x|-|y| \geq 0$

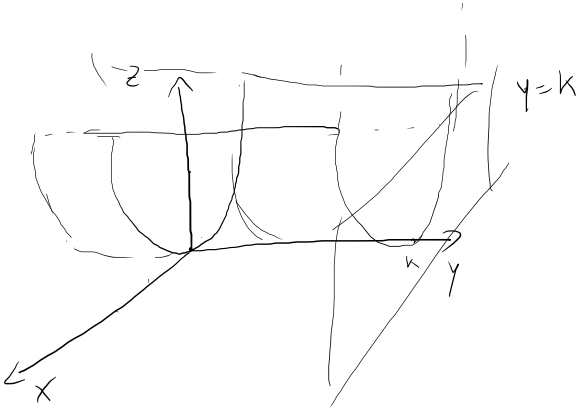


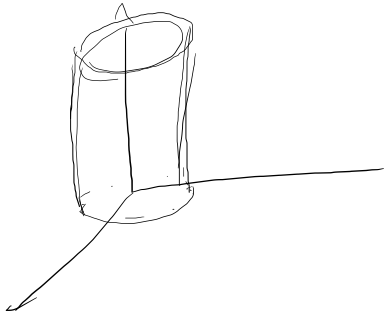
Načrtněte graf funkce

(b) $f(x,y) = x^2$ (parabolický válec),

$z = x^2$

$\begin{cases} z = x^2 \\ y = k \end{cases}$ $z = x^2$ $k \in \mathbb{R}$





$(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$

$C\ [a,b,c]$ $r = R$

(f) $f(x,y) = -\sqrt{4 - x^2 - y^2}$ (dolní polovina sféry),

$z = -\sqrt{4 - x^2 - y^2}$

$\Updownarrow$

$\begin{cases} z \le 0 \\ z^2 = 4 - x^2 - y^2 \end{cases}$ $\begin{cases} z \le 0 \\ x^2 + y^2 + z^2 = 4 \end{cases}$